

A little fiddling 'round resulted in the formula in the following PDF for the asymmetric trend filter weights that give exactly zero mean revision:



bias\_summ.pdf

The variance in the revision (from which the standard error can be computed) is given by:

$$\text{var}(R) = E(R^2) - [E(R)]^2,$$

but  $E(R) = 0$  by construction, so:

$$\text{var}(R) = E(R^2) \equiv \text{"mean square revision"}:$$

$$\begin{aligned} \text{var}(R) = & \left[ \sum_{i=1}^m (u_i - w_i)(\alpha + \beta i) \right]^2 + \sigma^2 \sum_{i=1}^m (u_i - w_i)^2 - 2 \sum_{i=1}^m \sum_{j=m+1}^n (u_i - w_i) w_j (\alpha + \beta i)(\alpha + \beta j) \\ & + \left[ \sum_{i=m+1}^n w_i (\alpha + \beta i) \right]^2 + \sigma^2 \sum_{i=m+1}^n w_i^2 \end{aligned}$$

where:

$u_i$  = asymmetric weights:  $i = 1, 2 \dots m$

$w_i$  = corresponding symmetric weights:  $i = 1, 2 \dots n$

$\alpha$  = intercept of linear regression fit to end of series

$\beta$  = slope of linear regression fit to end of series

$\sigma^2$  = intrinsic variance in series values (stationary)

$n$  = length of symmetric filter

$m$  = length of asymmetric filter. For a trend estimate at the  $k^{\text{th}}$  timepoint

from the end of a series ( $k = 1, 2 \dots (n-1)/2$ ; where  $k = 1 \Rightarrow$  endpoint),

$$\text{we have } m = \frac{(n-1)}{2} + k$$

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