Derivation of Equation (2) in:

[Subject: Estimating Generalised Proximity Effects under regARIMA (version 7.0)] :

The abovementioned equation defines a generalised proximity effect regressor, ie. a generic formulation of the explanatory variables to model this effect. We also refer the reader to the schematic in Figure 1 of the above document for the derivation below.

- We define the time dependence of the excess activity rate over the *"w"* ("before" moving holiday) period as:

$$\left(\frac{\mathrm{d}\mu}{\mathrm{d}t}\right)_{bef} = s_1 t^p , \qquad (1)$$

  where s_1 is a constant.

- We also define the time dependence of the excess activity rate over the *"h"* ("after/during" moving holiday) period:

$$\begin{aligned}\left(\frac{\mathrm{d}\mu}{\mathrm{d}t}\right)_{aft} &= \max\left[\left(\frac{\mathrm{d}\mu}{\mathrm{d}t}\right)_{bef}\right] - s_2 t^q \\ &= s_1 w^p - s_2 t^q , \qquad (2)\end{aligned}$$

  and where s_2 is also a constant.

- The "before" moving holiday explanatory variable, x_b , is given by the fraction of integrated activity in the *"w"* period that falls in the (reference) month prior to a period boundary (ie. that falls under the curve defined by eq. 1 upto some number of days *n*):

$$\begin{aligned}x_b &= \frac{\int_0^n s_1 t^p \mathrm{d}t}{\int_0^w s_1 t^p \mathrm{d}t} \\ &= \left(\frac{n}{w}\right)^{p+1} , \qquad (3)\end{aligned}$$

  where *n* = the number of w days falling in a "prior" (reference) month. This therefore completes the derivation of the "before" part of the regressor.

- By a similar construction, the "after/during" moving holiday explanatory variable, x_d , is given by the fraction of integrated activity in the *"h"* period that falls in the (reference) month prior to a period boundary (ie. that falls under the curve defined by eq. 2 upto some number of days *m*):

$$x_d = \frac{\int_0^m s_1 w^p - s_2 t^q \ \mathrm{d}t}{\int_0^h s_1 w^p - s_2 t^q \ \mathrm{d}t}$$

$$= \frac{s_1 w^p m - [1/(q+1)] s_2 m^{q+1}}{s_1 w^p h - [1/(q+1)] s_2 h^{q+1}}, \qquad (4)$$

where m = the number of h days falling in a "prior" (reference) month. We note that equation (2) must satisfy the following boundary condition at time $t = h$:

$$\left(\frac{\mathrm{d}\mu}{\mathrm{d}t}\right)_{aft(t=h)} = 0$$

$$\Rightarrow s_1 w^p - s_2 h^q = 0$$

$$\Rightarrow \frac{s_2}{s_1} = \frac{w^p}{h^q}. \qquad (5)$$

We now have an equation relating the unknown constants s_1 and s_2 that can be used for their elimination in equation (4). Performing this elimination and rearranging equation (4), we arrive at the final expression for the "after" part of the regressor:

$$x_d = \frac{\frac{m}{h} - \frac{1}{q+1}\left(\frac{m}{h}\right)^{q+1}}{1 - \frac{1}{q+1}}$$

$$= \left(\frac{m}{h}\right)\left[\frac{q+1-(m/h)^q}{q}\right]. \qquad (6)$$

This completes the derivation.