The model for the photometric error at plane i is

$$\varepsilon_i = \varepsilon_{pi} + \varepsilon_{ui} = \sum_{j=1}^{i} \Delta\varepsilon_{pj} + \varepsilon_{ui}$$

where ε_{pi} is the total photon noise at plane i, which is the sum of the incremental photon noise added at each plane leading up to plane i, $\Delta\varepsilon_{pj}$ at plane j, and ε_{ui} is the rest of the photometric error at plane i, assumed to be uncorrelated with all other errors in the ramp. We are interested in the expectation value of the product of the errors at any planes m and n:

$$\begin{aligned}\varepsilon_m \varepsilon_n &= \left(\sum_{j=1}^{m} \Delta\varepsilon_{pj} + \varepsilon_{um} \right) \left(\sum_{k=1}^{n} \Delta\varepsilon_{pk} + \varepsilon_{un} \right) \\ &= \sum_{j=1}^{m} \left(\Delta\varepsilon_{pj} \sum_{k=1}^{n} \Delta\varepsilon_{pk} \right) + \varepsilon_{um}\varepsilon_{un} + \varepsilon_{um} \sum_{k=1}^{n} \Delta\varepsilon_{pk} + \varepsilon_{un} \sum_{j=1}^{m} \Delta\varepsilon_{pj}\end{aligned}$$

Since ε_{um} is uncorrelated with all other errors in the ramp except for ε_{un} when $m = n$, the last two terms on the right will become zero when we take expectation values. Furthermore, each incremental photon-noise error $\Delta\varepsilon_{pj}$ is uncorrelated with each other $\Delta\varepsilon_{pk}$ except for $j = k$. For $m = n$, the expectation values are therefore

$$\left\langle \varepsilon_n^2 \right\rangle \equiv \sigma_n^2 = \left\langle \sum_{k=1}^{n} \Delta\varepsilon_{pk}^2 \right\rangle + \left\langle \varepsilon_{un}^2 \right\rangle = n\sigma_p^2 + \sigma_{un}^2$$

where σ_n^2 is the total photometric uncertainty at plane n, σ_p^2 is the incremental photon-noise uncertainty per plane (equal to the increase in DN per plane multiplied by the gain), and σ_{un}^2 is the uncertainty at plane n due to all effects other than photon noise. For $m \neq n$, let $m > n$; then

$$\left\langle \varepsilon_m \varepsilon_n \right\rangle \equiv \sigma_{mn}^2 = \left\langle \sum_{k=1}^{n} \Delta\varepsilon_{pk}^2 \right\rangle = n\sigma_p^2$$

The error covariance matrix for samples along the ramp can be constructed from these expressions once the value of σ_p^2 and all values of σ_{un}^2 are known. These can be approximated from the known DN values and their uncertainties. The average value of DN/plane can be multiplied by the gain to yield σ_p^2 (if nonlinearity effects are significant, some quadratic smoothing along the ramp or some other method could be used), and σ_{un}^2 is just the known value of σ_n^2 with $n\sigma_p^2$ subtracted from it (this should be clipped at some minimum value to protect against corrupted data).

The correlation coefficients corresponding to the covariances are

$$\rho_{mn} = \frac{\sigma_{mn}^2}{\sigma_m \sigma_n}$$

$$\rho_{mn} = \frac{n\sigma_p^2}{\sqrt{(m\sigma_p^2 + \sigma_{um}^2)(n\sigma_p^2 + \sigma_{un}^2)}}$$

To get a feel for how the correlation matrix might look, consider the case for which the uncorrelated error at each plane is three times as large as the incremental photon-noise error:

$$\sigma_{un}^2 = 3\sigma_p^2$$

$$\rho_{mn} = \frac{n}{\sqrt{(m+3)(n+3)}}$$

For a ten-plane ramp, the correlation-coefficient matrix would have the following values.

1.000									
0.224	1.000								
0.204	0.365	1.000							
0.189	0.338	0.463	1.000						
0.177	0.316	0.433	0.535	1.000					
0.167	0.298	0.408	0.504	0.589	1.000				
0.158	0.283	0.387	0.478	0.559	0.632	1.000			
0.151	0.270	0.369	0.456	0.533	0.603	0.667	1.000		
0.144	0.258	0.354	0.436	0.510	0.577	0.639	0.696	1.000	
0.139	0.248	0.340	0.419	0.490	0.555	0.614	0.669	0.721	1.000