The average result of Gaussian refinement of measurements with pure Poisson noise, averaged over the entire distribution except for n = 0 (which must be excluded because measurement results of 0 are taken as never occurring and would cause the inverse-variance weight to be infinite) is

$$\langle F \rangle = \frac{\sum_{n=1}^{\infty} p(n)\frac{F_n}{\sigma_n^2}}{\sum_{n=1}^{\infty} \frac{p(n)}{\sigma_n^2}} = \frac{\sum_{n=1}^{\infty} p(n)\frac{n}{n}}{\sum_{n=1}^{\infty} \frac{p(n)}{n}} = \frac{1 - p(0)}{\sum_{n=1}^{\infty} \frac{p(n)}{n}}$$

where $p(n)$ is the Poisson probability with mean λ:

$$p(n) = \frac{e^{-\lambda}\lambda^n}{n!}$$

and the inverse-variance weighting of the flux associated with Gaussian estimation is also weighted with the Poisson probability in order to average over the Poisson distribution. The summand in the denominator of the first equation is

$$\frac{p(n)}{n} = \frac{e^{-\lambda}\lambda^n}{nn!} = \frac{e^{-\lambda}\lambda^n}{(n+1)!}\left(\frac{n+1}{n}\right) = \frac{e^{-\lambda}\lambda^{n+1}}{(n+1)!}\left(\frac{n+1}{\lambda n}\right)$$

Substituting $k = n+1$, the denominator becomes

$$\sum_{k=2}^{\infty} \frac{e^{-\lambda}\lambda^k}{k!}\left(\frac{k}{\lambda(k-1)}\right) = \sum_{k=2}^{\infty} p(k)\left(\frac{k}{\lambda(k-1)}\right) \approx \left\langle \frac{k}{\lambda(k-1)} \right\rangle \approx \frac{\lambda}{\lambda(\lambda-1)} = \frac{1}{\lambda-1}$$

where the first approximation is inexact because the first two terms in a normal Poisson-weighted average are omitted, and the second approximation assigns the mean of the distribution to the approximately averaged value of the index k. Now the first equation becomes

$$\langle F \rangle \approx \frac{1 - p(0)}{\frac{1}{\lambda-1}} = (\lambda-1)(1 - p(0)) \approx \lambda - 1$$

This approximation has been found to very good for $\lambda > 100$ and is accurate to 12% for $\lambda = 10$.