Instructions: Midterm is issued at the end of class on Monday, April 4. It is due at the beginning of class (i.e., 4pm) on Wednesday, April 6.

You must work independently. You are bound by your honor not to consult anyone about this midterm, but you may use any other resource you wish in preparing your answers. Attempt all problems, and show all of your work.

Please state clearly any assumptions you are making, or any approximations you are using. Show all steps in your calculations and justify any expressions, formulae that you use in your calculations.

Updates & Help: You may contact the instructors for clarification on any of the problems. Any hints, additional information, corrections, etc., given in response will be made available to the entire class on the class midterm webpage at:

http://spider.ipac.caltech.edu/staff/squires/classes/astr420_2005a/midterms/midterm_02.html

It is recommended that you check this webpage frequently for updates.
Problem 1: Photon Propagation in a Closed Universe (10 pts)

a. Consider a universe where $k = 0, -1, 1$ corresponds to flat, open and closed universes, respectively, and the FRW metric is:

$$ds^2 = cdt^2 - R^2(t) \left[ \frac{d\omega^2}{1 - k\omega^2} + \omega^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

Show that this can be re-expressed as

$$ds^2 = cdt^2 - R^2(t) \left[ d\chi^2 + \Sigma^2(\chi)(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$  \hspace{1cm} (1)

or as

$$ds^2 = R^2(\eta) \left[ d\eta^2 - d\chi^2 - \Sigma^2(\chi)(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$  \hspace{1cm} (2)

where $\Sigma^2(\chi) = \sin^2(\chi), \chi^2, \sinh^2 \chi$ for $k = 1, 0, -1$, respectively.

b. Solve the first order Friedmann equation

$$\left( \frac{dR/dt}{R} \right)^2 = \frac{8\pi G \rho}{3} - \frac{k c^2}{R^2}$$  \hspace{1cm} (3)

for a closed, $k=1$, matter dominated universe.

c. For a closed, $k=1$, universe in which radiation dominates for only a negligibly short fraction of time, how many times can a light ray travel around the universe from the moment of creation to the moment when it recollapses towards a singularity? \([\text{Hint: use your solutions to 1a, b.}]\)
Problem 2: Clusters of Galaxies (10 pts)

Consider a fictional cluster of galaxies, known colloquially as *El Gordo*, at $z = 0.05$.

a. In an isotropic, homogeneous universe with $\Lambda = 0$ and $H_0 = 70$ km/s/Mpc, what is the proper distance to this cluster from Earth [in Mpc]? Consider the cases of $k = 0, \pm 1$, corresponding to $\Omega_0 = 1, 1.5, \text{ and } 0.2$, respectively.

b. For a $k = 0$ universe, how long ago was a light ray emitted from a galaxy in *El Gordo* if we observe it today? To what fraction of the Hubble time does this correspond? What fraction of the Hubble time had elapsed when this photon was emitted?

c. Assume the galaxy radial velocity dispersion in *El Gordo* is 1000 km/s. Estimate how long it would take for a galaxy to travel to from one side of the cluster to the other. Assume that the galaxy travels with a speed equal to the cluster’s radial velocity dispersion, and that *El Gordo* is of similar physical size as Coma. How does this compare with the Hubble time? What does this imply for the dynamical state of *El Gordo*?
Problem 3: Superluminal Motion (10 pts)

In Chapter 26, we derived equation 26.10:

\[ \frac{v}{c} = \frac{v_{\text{app}}/c}{\sin \phi + (v_{\text{app}}/c) \cos \phi}, \]

which relates the apparent velocity \( v_{\text{app}} \) of a source moving at velocity \( v \) at angle \( \phi \) relative to the observer’s line of sight. Here we will derive equations 26.12 and 26.13, left as exercises in the text, which describe the extreme values of \( v \) as a function of \( \phi \).

a. Derive equation 26.13. \([ Hint: \text{the extrema of } v/c \text{ as a function of } \phi \text{ will occur when } d(v/c)/d\phi = 0. ]\]

b. Derive equation 26.12. \([ Hint: \cot \phi = \frac{\cos \phi}{\sin \phi} = \frac{\sqrt{1-\sin 2\phi}}{\sin \phi}. ]\)
Problem 4: Distance Scale with type Ia SNe (10 pts)

Type Ia supernovae are an excellent distance indicator: the peak brightness is a so-called “standard candle”. That is, at maximum brightness, all type Ia have an absolute magnitude of $M_B = -19.6 \pm 0.2$. Therefore, knowing the absolute magnitude at peak brightness, and measuring the apparent magnitude allows one to solve for the distance.

a. Starting from the relationship

$$d = 10^{(m-M-a+5)/5}$$

derive a relation between $(m - M)$ and log($z$), for small redshift and $H = 100 \, h \, \text{km/s/Mpc}$.

b. Given the plot of apparent - absolute peak magnitude, $(m-M)$, versus redshift, $z$, for the following type Ia supernovae, what is the Hubble constant in the following universe? You may assume magnitudes have been extinction corrected.
Problem 5: Short Answers

a. What is one observational challenge of studying gamma-ray bursts? (2 pts)

b. The galaxy ngc 0024 has a heliocentric recessional velocity of 554 km/s, and a distance of \(8.2 \, h_{0,7}^{-1} \, \text{Mpc}\). Compare and explain. (2 pts)

c. What is the present value of the Hubble constant in the following universe (3 pts):

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>distance [Mpc]</th>
<th>recessional velocity [km/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>340</td>
</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>0.5</td>
<td>35</td>
</tr>
<tr>
<td>D</td>
<td>15</td>
<td>1050</td>
</tr>
<tr>
<td>E</td>
<td>50</td>
<td>3550</td>
</tr>
<tr>
<td>F</td>
<td>95</td>
<td>6700</td>
</tr>
<tr>
<td>G</td>
<td>35</td>
<td>2415</td>
</tr>
<tr>
<td>H</td>
<td>35</td>
<td>2520</td>
</tr>
<tr>
<td>I</td>
<td>51</td>
<td>3470</td>
</tr>
</tbody>
</table>

d. The Lyman-\(\alpha\) transition of hydrogen, at a wavelength of 1216 Å, corresponds to electrons shifting between the two lowest energy states in a hydrogen atom and can be one of the most powerful lines in extragalactic sources. The optical window of the electromagnetic spectrum, defined by the opacity of the terrestrial atmosphere, ranges from 3500 Å to 9500 Å. Find the corresponding redshift range over which Lyman-\(\alpha\) is visible at optical wavelengths. (2 pts)

e. What is the difference between the broad-line region and the narrow-line region of active galaxies? (2 pts)

f. Explain the difference between a quasar and Seyfert galaxy. (2 pts)

g. Significant evidence shows that quasars were considerably more common in the early universe compared with the present-day universe. Where are these quasars now? (2 pts)